INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.MATH - Third Year, 2014-15
Statistics - III, Backpaper Examination, January 6, 2015
Marks are shown in square brackets. **Total Marks: 50**

1. Let $\mathbf{Y} \sim N_n(\mathbf{0}, \sigma^2 I_n)$. Find the conditional distribution of $\mathbf{Y}'\mathbf{Y}$ given $\mathbf{a}'\mathbf{Y} = 0$ where $\mathbf{a}$ is a non-zero constant vector. [10]

2. Consider the model $\mathbf{Y} = \mathbf{X}\beta + \epsilon$, where $\mathbf{X}_{n\times p}$ has $\mathbf{1}$ as its first column and $\epsilon \sim N_n(\mathbf{0}, \sigma^2 I_n)$.
(a) If $\hat\beta$ is the least squares estimator of β, show that $(\hat\beta - \beta)'\mathbf{X}'\mathbf{X}(\hat\beta - \beta)$ is distributed independently of the residual sum of squares.
(b) Consider the case when there is only one regressor, X_1. When do we have independence of $\hat\beta_0$ and $\hat\beta_1$?
(c) Find the maximum likelihood estimator of σ^2. Is it unbiased? [10]

3. Consider the following model:
$y_1 = \theta + \gamma + \epsilon_1$
$y_2 = \theta + \phi + \epsilon_2$
$y_3 = 2\theta + \phi + \gamma + \epsilon_3$
$y_4 = \phi - \gamma + \epsilon_4$,
where ϵ_i are uncorrelated having mean 0 and variance σ^2.
(a) Show that $\gamma - \phi$ is estimable. What is its BLUE?
(b) Find the residual sum of squares. What is its degrees of freedom? [10]

4. Let $\mathbf{X} = (X_1, X_2, X_3, X_4)'$ have mean $\mathbf{0}$ and covariance matrix $\sigma^2\left\{(1-a^2)I_4 + a^2\mathbf{1}\mathbf{1}'\right\}$, for some $0 < |a| < 1$ and where $\mathbf{1}$ is the vector with all elements equal to 1. Find the partial correlations, $\rho_{12.3}$ and $\rho_{12.34}$. [10]

5. What is a 2-factor 2-way ANOVA model? Derive its AVOVA table. Give an expression for the coefficient of determination for such a model. [10]